Conclusions

Insofar as the statements of geometry speak about reality, they are not certain, and insofar as they are certain, they do not speak about reality. (Einstein, 1921, p. 3.)

Fractal geometry has emerged in direct response to the need for better mathematical descriptions of reality, and there is little doubt that it provides a powerful tool for interpreting and rendering natural systems. Yet in its wake has come, once again, the realization that all knowledge is contingent upon its context in time and space, that good theory is relative to what we already have and have had before, thus reminding us of Einstein's (1921) thoughts on the limitations of any geometry, indeed of all mathematics. Although extending our abilities to model both natural and artificial systems, fractals impress even further upon us the inherent complexity and uncertainty of the world we live in. In this sense, one kind of uncertainty – that involving the inapplicability of Euclidean geometry to many real systems – has been replaced with another – a more appropriate geometry for simulating reality, but one which is based on the notion that reality itself has infinite complexity in the geometric sense.

In this conclusion, we will attempt to pull the diverse threads which we have woven in this book together, and suggest directions in which the application of fractal geometry to cities as well as the theory of the fractal city might develop. Throughout, we have made many suggestions and identified many problems, all of these being worthy of further research, and we will not attempt to list these again. What we will do, is summarize the theory as it has emerged here, thus providing readers with both a sense of closure as well as some directions in which we feel this work should be taken further. In one sense, we can see this book in two parts: first in the early chapters, we presented the rudiments of fractal geometry and mildly suggested ways in which it might pertain to the physical form or morphology of cities. In the second part, from Chapter 5 onwards, we argued that the city itself is fractal and the new geometry the obvious medium for its measurement and simulation. In this second part, we also drew a major distinction between fractals as applied to single cities and to systems of cities, to intra-urban and to inter-urban spatial structure. But our exposition has been mainly from the standpoint of fractals as they are applicable to cities, and not the other way around. Perhaps it is time to change and rework the edifice of urban spatial theory, noting the ways in which fractals arise naturally and spontaneously, once we now have this new geometry in place. This has not been our quest here, but doubtless in time, the map will be completed in this way by others.

We are now able to provide a reasonably coherent summary of the ways in which fractal geometry is applicable to cities in general, urban growth and form in particular, but before we do so, a word about dimension. The concepts of Euclidean dimension are so deeply ingrained that we use them and will continue to do so as a shorthand to describe the magnitude and complexity of many systems of interest. For example, notwithstanding the fact that we now know that the dimension of any real system is fractional, we still refer to it as existing in n dimensions where n is an integer (usually the integral part of the fractal dimension). This is important here in that we can articulate cities as having properties that can still be measured as points, lines, areas and volumes, from zero to three dimensions, or even beyond if our geometry is one that results from urban processes which can be visualized in mathematical space. However, in this context, most of our ideas are based on conceiving of the city as sets of lines and areas, and thus our geometry is based upon one and two dimensions, not zero or three, although there are arguments which suggest that cities might be treated as points or volumes, thus composing fruitful extensions to the new geometry.

The way we have represented the geometry of the city has been central to our analysis. In essence, cities are conceived as filling two-dimensional space, as sets of connected points and lines which form areas, less than the entire space in which they might exist but more than simply the straight line; their fractal dimensions must therefore fall between one and two. It was only in Chapter 7 that we began to treat cities in this way for we first introduced a simplification to the geometry, approximating areas as boundaries which we dealt with in Chapters 5 and 6. In short, we proposed that a growth model for the city based on diffusion-limited aggregation (DLA) with a dimension $D \approx 1.71$, represents an idealized model of the way in which urban space is filled, while a simplified form for the boundary of the space filled was based on the Koch curve with a dimension $D \approx 1.26$. We did not provide a rigorous link between DLA and the Koch curve, but we did present sufficient examples to show that the dimensions of idealized and real boundaries are less in value than those for the entire cities from which they are formed.

In developing fractal geometry, we introduced two methods for deriving dimensions, the first based on changing the scale over which an object is measured, the second based on changing its size. We mainly used the first method for urban boundaries although it can be used for areas (Batty and Xie, 1994), whereas the second method is appropriate to systems where we can grow the city into the space which it fills. In another sense, our distinction between boundaries and areas filled is one between treating the city in static as opposed to dynamic terms, the Koch model being a static model of the way scale is varied, the DLA model being a dynamic one where the object is grown by varying its size. The relations between the object, the city, and scale and size are generally the same in that population N, measured by the number of elements composing the urban boundary or space filled, is related to scale r or size R through power laws involving the fractal dimension D. All subsequent analysis flows from these premises.

The critical relations for boundaries relate number of elements and their length to scale but there are few substantive implications for urban theory.

For growing cities, however, the link to urban theory is much stronger and more suggestive. Population as a function of linear size is easily generalizable to area which is at the basis of allometry, the study of relative size, while once area is invoked, density can be defined. This relates the entire analysis to mainstream urban economics where classic density profiles for cities are outcomes of diverse market clearing processes based on the conventional micro-economic behavior of the land market. In short, the key relations for the city which involve fractal dimension, relate population and its density to linear size and area, these relations being structured in incremental or cumulative form. Moreover, we showed in Chapter 9 that these relations appear to have greater rationale than those used traditionally, and what is more, that the whole approach shows how careful one must be in defining and measuring densities. One conclusion is that much of the work on urban density theory and its applications over the last 40 years should be reworked in the light of these developments.

There are many extensions to this geometry which we have pondered since we began this work. The obvious one which we have explored in part elsewhere, involves growing cities based on more than one seed or center, that is moving from a monocentric to a multicentric context. We explored the influence of two cities planted from separate seeds growing towards one another, thus forming a larger urban aggregate in terms of the consequent mixing of dimensions (Fotheringham, Batty and Longley, 1989), but we barely touched the surface of these ideas, and there is all still to be done. We have also begun to explore DLA in three dimensions, and to speculate on what a three-dimensional urban fractal might be like, but so far, we have not had the resources to pursue this line of attack to any conclusion. We have explored many modifications to the growth processes in DLA-like models which give rise to different urban forms, hence fractal dimensions, we have mixed processes and dimensions, and we have considered ways in which our growth models might incorporate reversibility. However, we have but scratched the tip of an iceberg, and to extract even the smallest kernel of knowledge which will advance our understanding of urban form, there is an enormous research program to initiate.

Extending fractal geometry to systems of cities is comparatively straightforward. Hints have been provided in Chapters 1 and 10, but a thorough analysis is yet to be attempted. We have shown how the central place hierarchy is fractal as evidenced by the rank-size distribution, and we have speculated that population densities must fall, and fractal dimensions increase as cities move up their hierarchy. But we have not shown how this possibility is consistent with the growth of the single city and its size; for the analysis of a single growing city implies nothing about the way cities might grow and compete within a hierarchy. In this book, our analysis has been largely confined to the single city, to intra-urban spatial structure, and extensions to systems of cities must therefore be high on any research agenda.

We began this book by examining visual perceptions of urban form, the traditional starting point for understanding the city. Indeed, our initial forays into the geometry of cities were in terms of how we might render their form through data and models so that we might generate more realistic and more communicable pictures using computer graphics. This somewhat serendipitous approach (which incidentally has been largely responsible for the general awakening of interest in fractals) did, however, introduce the idea that computers are laboratories for visualizing urban form, with great potential to enhance our understanding as well as to communicate complex ideas in manageable form. Throughout, we have been intent upon developing computer models which can ultimately be used, in such laboratory settings, to visualize different urban forms, with very different degrees of realism and prospects for realization. Now we have come to the end, we must admit that our models are still highly simplistic, yet do contain the rudiments of reasonable explanation, particularly those which we examined in Chapter 8.

The question some will ask is whether or not these ideas have any relevance for real policy making and planning. The answer we must give is, of course, contingent upon context, but we would argue that these ideas are as relevant in thinking about current urban problems such as energy, transportation, spatial polarization and segregation, planning control and so on as those currently advocated. But they are certainly less accessible, although our quest has been to make them a little more so and computer graphics is central to this. What fractal geometry does establish is that cities like most other real systems manifest a myriad of infinite complexity and this must change our responses to urban planning which have hitherto been simplistic and unrealistic, to say the least. Barnsley (1988a) who we quoted at the beginning of this book, says that "Fractal geometry will make you see everything differently", but it also changes our perceptions concerning the certainty of the reality and how we might manipulate it. There is now renewed hope that we might be able to forge a more conclusive link between the physical form of cities and the various social, economic and institutional processes that are central to their functioning.